Nuclear Effects in Deep Inelastic Scattering of Polarized Electrons off Polarized ³He and the Neutron Spin Structure Functions

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Abstract

It is shown that the nuclear effects playing a relevant role in Deep Inelastic Scattering of polarized electrons by polarized ³He are mainly those arising from the effective proton and neutron polarizations generated by the S' and D waves in ³He. A simple and reliable equation relating the neutron, g_1^n, and ³He, g_1^3, spin structure functions is proposed. It is shown that the measurement of the first moment of the ³He structure function can provide a significant check of the Bjorken Sum Rule.

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The spin structure functions (SSF) of the nucleon g_1^N and g_2^N provide information on the spin distribution among the nucleon partons and can allow important tests of various models of hadron's structure [1]. Our experimental knowledge is limited at present to the proton SSF g_1^p [2,3] and it is for this reason that new experiments [4–6] are under way aimed at improving the knowledge of g_1^p, as well as at measuring, for the first time, the proton SSF g_2^p and the neutron SSF g_1^n and g_2^n. The latter quantities are expected to be obtained from the spin asymmetry measured in Deep Inelastic Scattering (DIS) of longitudinally polarized electrons off polarized nuclear targets, viz $^2\vec{\text{H}}$ and $^3\vec{\text{He}}$. As is well known, the use of $^3\vec{\text{He}}$ targets, which will be considered in this paper, is motivated by the observation that, in the simplest picture of ^{3}He (all nucleons in S wave), protons have opposite spins so that their contribution to the asymmetry largely cancels out. However, such a cancellation does not occur if other components of the three body wave function are considered; moreover, the fact that electrons scatter off nucleons having a certain momentum and energy distribution may, in principle, limit the possibility to obtain information on nucleon SSF from scattering experiments on nuclear targets. The aim of this Rapid Communication is to quantitatively illustrate whether and to which extent the extraction of g_1^n from the asymmetry of the process $^3\vec{\text{He}}(\vec{e}, e')$X could be hindered by nuclear effects arising from small wave function components of ^{3}He, as well as from Fermi motion and binding correction effects on DIS. To this end, we use the spin dependent spectral function of ^{3}He [7], which allows one to take into account at the same time Fermi motion and binding corrections, unlike previous calculations [8] where only Fermi motion effects were considered. It should be pointed out that our paper is also based on a recent, improved description of inclusive scattering of polarized electrons by polarized nuclei [9,10], which leads in the quasi elastic kinematics to appreciable differences with respect to previous calculations [7,11]; therefore we will also check whether these differences persist in the DIS region.

In the Bjorken limit the longitudinal asymmetry for inclusive scattering of longitudinally polarized electrons off a polarized $J = \frac{1}{2}$ target with atomic weight A, reads as follows:

$$A_{\parallel} = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}} = 2x\frac{g_1^A(x)}{F_2^A(x)} \equiv A_{\vec{A}} \tag{1}$$

where $\sigma_{\uparrow\uparrow(\uparrow\downarrow)}$ is the differential cross section corresponding to the target spin parallel (antiparallel) to the electron spin, $x = Q^2/2M\nu$ is the Bjorken variable, g_1^A and F_2^A are the nuclear spin-dependent and spin–independent structure functions of the target A. In what follows, three models for the asymmetry, in order of increasing complexity, will be considered, *viz*:

1) *No nuclear effects.* This model is such that the following equations hold:

$$g_1^3(x) = g_1^n(x) \tag{2}$$

$$A_{^3\vec{\text{He}}} = f_n A_{\vec{n}} \tag{3}$$

where $A_{\vec{n}}(x) = 2xg_1^n(x)/F_2^n(x)$ is the neutron asymmetry and $f_n = F_2^n(x)/(2F_2^p(x)+F_2^n(x))$ the neutron dilution factor. Such a picture is equivalent to consider polarized electron scattering off $^3\vec{\text{He}}$ described as a pure symmetric S wave disregarding, moreover, Fermi motion and binding effects.

2) *Proton contribution within realistic wave function of ^3He.* Besides the S wave, the three body wave function contains a percentage of S' and D waves, $P_{S'}$ and P_D, which are responsible for a proton contribution to the polarization of $^3\vec{\text{He}}$. The amount of such a contribution can be calculated by considering the quantities $P_{p(n)}^{(\pm)}$, representing the probability to have a proton (neutron) with spin parallel $(+)$ or antiparallel $(-)$ to ^{3}He spin. In a pure S wave state $P_n^{(+)} = 1$, $P_n^{(-)} = 0$ and $P_p^{(+)} = P_p^{(-)} = \frac{1}{2}$, whereas for a three–body wave function containing S, S' and D waves, one has [12,13] $P_n^{(+)} = 1 - \Delta$, $P_n^{(-)} = \Delta$, $P_p^{(\pm)} = \frac{1}{2} \mp \Delta'$, where $\Delta = \frac{1}{3}[P_{S'} + 2P_D]$ and $\Delta' = \frac{1}{6}[P_D - P_{S'}]$. From world calculations on the three body system one obtains, in correspondence of the experimental value of the binding energy of ^{3}He, $\Delta = 0.07 \pm 0.01$ and $\Delta' = 0.014 \pm 0.002$ [12]. If the S' and D waves are considered and Fermi motion and binding effects are disregarded, one can write:

$$g_1^3(x) = 2p_p g_1^p(x) + p_n g_1^n(x) \tag{4}$$

$$A_{^3\vec{\text{He}}} = 2f_p p_p A_{\vec{p}} + f_n p_n A_{\vec{n}} \tag{5}$$

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where $f_{p(n)} = F_2^{p(n)}/(2F_2^p + F_2^n)$ is the proton (neutron) dilution factor, $A_{\vec{p}(\vec{n})} = 2x g_1^{p(n)}/F_2^{p(n)}$ is the proton (neutron) asymmetry and the effective nucleon polarizations are:

$$p_p = P_p^{(+)} - P_p^{(-)} = -0.028 \pm 0.004 \tag{6}$$

$$p_n = P_n^{(+)} - P_n^{(-)} = 0.86 \pm 0.02 \tag{7}$$

3) *Proton contribution within the convolution approach.* In order to take into account Fermi motion and binding effects, we have extended to polarized DIS the usual convolution approach adopted to treat the unpolarized DIS [14]. Let us first consider the general case of inclusive scattering by spin $\frac{1}{2}$ targets in impulse approximation. We obtain for the nuclear spin structure function g_1^A the following expression:

$$
\begin{aligned}
g_1^A(x, Q^2) = \sum_N \int dz \int dE \int d\mathbf{p} \Bigg\{ & \frac{1}{z} g_1^N \left(\frac{x}{z}, Q^2 \right) \left[P_{||}^N(\mathbf{p}, E) + \left(\frac{p_{||}}{E_p + M} - \frac{\nu}{|\mathbf{q}|} \right) \frac{|\mathbf{p}|}{M} \mathcal{P}^N(\mathbf{p}, E) \right] \\
& - C \frac{Q^2}{|\mathbf{q}|^2} \frac{1}{z} g_1^N \left(\frac{x}{z}, Q^2 \right) \frac{1}{M} \left[\frac{|\mathbf{p}|^2}{2(E_p + M)} \Phi(\alpha) \mathcal{P}^N(\mathbf{p}, E) + \frac{M}{2} \frac{\Phi(\alpha)}{\sin\alpha} P_\perp^N(\mathbf{p}, E) \right] \\
& + C \frac{Q^2}{|\mathbf{q}|^2} \frac{1}{z^2} g_2^N \left(\frac{x}{z}, Q^2 \right) \frac{1}{M} \left[\frac{|\mathbf{p}|^2}{2(E_p + M)} \Phi(\alpha) \mathcal{P}^N(\mathbf{p}, E) - \frac{E_p}{2} \frac{\Phi(\alpha)}{\sin\alpha} P_\perp^N(\mathbf{p}, E) \right. \\
& \left. - \frac{|\mathbf{q}|}{\nu} p_{||} \left(P_{||}^N(\mathbf{p}, E) - \frac{P_\perp^N(\mathbf{p}, E)}{\tan\alpha} \right) \right] \Bigg\} \delta \left(z - \frac{p \cdot q}{M\nu} \right)
\end{aligned}
\tag{8}
$$

where $p \equiv (p^0, \mathbf{p})$ is the four–momentum of the bound nucleon, with $p^0 = M_A - [(E - M + M_A)^2 + |\mathbf{p}|^2]^{\frac{1}{2}}$; E is the nucleon removal energy; $E_p = [M^2 + |\mathbf{p}|^2]^{\frac{1}{2}}$; $\Phi(\alpha) = (3\cos^2\alpha - 1)/\cos\alpha$, with $\cos\alpha = \mathbf{p} \cdot \mathbf{q}/|\mathbf{p}||\mathbf{q}|$; $p_{||} = |\mathbf{p}|\cos\alpha$; C is a constant to be discussed later on; $P_{||}^N(\mathbf{p}, E)$, $P_\perp^N(\mathbf{p}, E)$, $\mathcal{P}^N(\mathbf{p}, E)$ are defined as follows [7]:

$$P_{||}^N(\mathbf{p}, E) = P_{\frac{1}{2}\frac{1}{2}M}^N(\mathbf{p}, E) - P_{-\frac{1}{2}-\frac{1}{2}M}^N(\mathbf{p}, E) \tag{9}$$

$$P_\perp^N(\mathbf{p}, E) = 2 P_{\frac{1}{2}-\frac{1}{2}M}^N(\mathbf{p}, E) e^{i\phi}, \tag{10}$$

$$\mathcal{P}^N(\mathbf{p}, E) = \sin\alpha P_\perp^N(\mathbf{p}, E) + \cos\alpha P_{||}^N(\mathbf{p}, E) \tag{11}$$

where ϕ is the polar angle, and

$$
\begin{aligned}
P_{\sigma\sigma'M}^N(\mathbf{p}, E) = \sum_f & [\langle \psi_{A-1}^f; N, \mathbf{p}, \sigma' | \psi_{J,M} \rangle]^* \\
& [\langle \psi_{A-1}^f; N, \mathbf{p}, \sigma | \psi_{J,M} \rangle] \\
& \delta(E - E_{A-1}^f + E_A) \ .
\end{aligned}
\tag{12}
$$

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is the spin dependent spectral function. Of particular relevance are the "up" and "down" spectral functions $P^N_{\frac{1}{2}\frac{1}{2}\frac{1}{2}}$ and $P^N_{-\frac{1}{2}-\frac{1}{2}\frac{1}{2}}$, respectively, for they determine the effective nucleon polarization, *viz*:

$$P_N^{(+)} = \int P^N_{\frac{1}{2}\frac{1}{2}\frac{1}{2}}(\mathbf{p}, E) d\mathbf{p} dE \tag{13}$$

$$P_N^{(-)} = \int P^N_{-\frac{1}{2}-\frac{1}{2}\frac{1}{2}}(\mathbf{p}, E) d\mathbf{p} dE \ . \tag{14}$$

Using in Eq. (8) the proper nucleon SSF $g^N_{1(2)}$, the nuclear SSF g^A_1 can be evaluated in the quasi elastic, inelastic and DIS regions. Two different prescriptions were used up to now to obtain the convolution formula: the one of Ref. [11] (to be called prescription 1), corresponding to $C = 0$ in Eq. (8) (such a convolution formula has also been used in Ref. [7] where binding effects in quasi–elastic scattering have been investigated), and the one of Ref. [9] (to be called prescription 2) corresponding to $C = 1$. The theoretical soundness of both prescriptions, in particular some drawbacks of prescription 1, as well as their impact on the quasi–elastic asymmetry, have been discussed in Ref. [9] and in Ref. [10], and shall not be repeated here; the important point to be stressed, in the context of the present investigation, is that in the Bjorken limit ($\nu/|\mathbf{q}| \rightarrow 1$, $Q^2/|\mathbf{q}|^2 \rightarrow 0$) both of them lead to the same result, namely:

$$g^A_1(x) = \sum_N \int_x^A dz \frac{1}{z} g^N_1\left(\frac{x}{z}\right) G^N(z) \ , \tag{15}$$

with the spin dependent light cone momentum distribution given by:

$$G^N(z) = \int dE \int d\mathbf{p} \left\{ P^N_{\parallel}(\mathbf{p}, E) - \left[1 - \frac{p_{\parallel}}{E_p + M}\right] \frac{|\mathbf{p}|}{M} \mathcal{P}^N(\mathbf{p}, E) \right\} \delta\left(z - \frac{p^+}{M}\right) \ , \tag{16}$$

where $p^+ = p^0 - p_{\parallel}$ is the light cone momentum component. We see that g^A_1 depends only upon g^N_1, whereas it turns out [15] that g^A_2 depends both on g^N_1 and g^N_2.

In our calculations, the nucleon SSF g^N_1 is the one proposed in Ref. [16], representing an extension of the Carlitz–Kaur model [17] by allowing spin dilution of the valence quark due to gluon polarization;the effective nucleon polarization $p_{p(n)}$ are given by Eqs. (6) and (7); the spin dependent spectral functions are the ones obtained in Ref. [7], yielding values of $p_{p(n)}$ (cf. Eqs. (13) and (14)) in agreement with (6) and (7).

The $^3\vec{\text{He}}$ asymmetry (Eq. (1)) calculated using the convolution formula for g_1^3 (Eq. (15)) and the corresponding formula for the unpolarized structure function F_2^3 (see Ref. [14]) is presented in Fig.1(a), and the nuclear structure function g_1^3 is shown in Fig.1(b). The general trend of our results resembles the one found in Ref. [8], except for the asymmetry at $x > 0.9$ and g_1^3 at $x \simeq 0$. We will discuss the origin of these differences later on; now we would like to stress the following point: the non vanishing proton contribution to the asymmetry shown in Fig.1(a) hinders in principle the extraction of the neutron structure function from the $^3\vec{\text{He}}$ asymmetry. As a matter of fact, once g_1^3 is obtained from the experimental asymmetry, the theoretically estimated proton contribution $g_1^{3,p}$ has to be subtracted from it in order to obtain the neutron contribution $g_1^{3,n}$. It can be seen from Fig.1(b) that for $0.01 \leq x \leq 0.3$ this quantity differs from the neutron structure function g_1^n by a factor of about 10%; since this factor is generated by nuclear effects, one might be tempted to consider it as the theoretical error on the determination of g_1^n; however, it should be remembered that the difference between g_1^n and $g_1^{3,n}$ is in principle model dependent through the way nuclear effects are introduced and the specific form of g_1^n used in the convolution formula. Thus it is necessary to understand the origin of the nuclear effects and how much they depend upon the form of g_1^N. To this end, the asymmetry and the structure function predicted by the convolution approach are compared in Fig.2 with the predictions of the simpler models represented by Eqs. (2)–(5). It can be seen that the model which completely disregards nuclear effects (binding and Fermi motion as well as S' and D waves), predicts an asymmetry which strongly differs from the ones which include nuclear effects; however it can also be seen that at least for $x \leq 0.9$ nuclear effects can reliably be taken care of by Eq. (5), i.e. by considering that the only relevant nuclear effects are due to the effective nucleon polarization induced by S' and D waves. Such a conclusion is very clearly demonstrated in Fig.3, where the free neutron structure function is compared with the quantity (cf. Eq. (4)):

$$\tilde{g}_1^n(x) = \frac{1}{p_n} \left[g_1^3(x) - 2p_p g_1^p(x) \right] \tag{17}$$

calculated using the convolution formula for $g_1^3(x)$; it can be seen that the two quantities are

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very close to each other, differing, because of binding and Fermi motion effects, by at most 4%. Such a small difference can be understood by expanding $\frac{1}{z}g_1^N\left(\frac{x}{z}\right)$ in Eq. (15) around $z = 1$ and by disregarding the term proportional to $\mathcal{P}^N$ in Eq. (16), which gives anyway a very small contribution being of the order $|\mathbf{p}|/M$; one obtains [15]:

$$g_1^{3,N}(x) \sim g_1^N(x)\left(p_N + \frac{\Lambda_N}{M}\right) + x\frac{dg_1^N(x)}{dx}\frac{\Lambda_N}{M} + \dots \tag{18}$$

where $\Lambda_N = \left[\langle E_N\rangle^{(+)} + \langle T_{A-1}^N\rangle^{(+)} - (\langle E_N\rangle^{(-)} + \langle T_{A-1}^N\rangle^{(-)})\right]$, $\langle E_N\rangle^{(\pm)}$ and $\langle T_{A-1}^N\rangle^{(\pm)}$ being the average removal and recoil energies in the "up" and "down" states. Note that the difference between these quantities appearing in Λ_N results from the very definition of the polarized spectral function $P_\parallel^N$ (cf. Eq. (9))(in unpolarized DIS, which is governed by the unpolarized spectral function defined as the sum of the "up" and "down" spectral functions, the difference in Λ_N is replaced by a sum [14]). Using the values of $\langle E_N\rangle^{(\pm)}$ and $\langle T_{A-1}^N\rangle^{(\pm)}$ resulting from three body realistic calculations [7], one gets $\Lambda_n/M \sim 0.72\cdot 10^{-3}$ and $\Lambda_p/M \sim 0.25\cdot 10^{-3}$, so that the first term of Eq. (18) yields Eq. (4) and the second term, representing Fermi motion and binding corrections, yields only a few percent contribution up to $x \sim 0.7$. Thus we have theoretically justified the correctness of Eq. (4) and demonstrated that the smallness of Fermi motion and binding is rather independent of the form of any well behaved g_1^N, for large variations of $dg_1^N(x)/dx$ are killed anyway by the smallness of Λ_N. To sum up, we have shown that the only relevant nuclear effects in inclusive DIS of polarized electrons off polarized ^{3}He, are those related to the proton and neutron effective polarizations arising from S' and D waves, and that such a result does not crucially depend upon the form of g_1^N. Therefore, the neutron structure function can be obtained from the $^3\vec{\text{He}}$ asymmetry using in Eq. (17) the experimental values for g_1^3 and g_1^p and the theoretical quantities p_p and p_n; the resulting theoretical errors due to Fermi motion and binding (about 5%) and to the uncertainties on p_p and p_n (cf. Eqs. (6) and (7)), lead to a total error well below than hitherto assumed [5,6]. The differences between our results and the ones of Ref. [8] previously mentioned, are also clear: the value of the proton polarization generated by the wave function used in Ref. [8] is $p_p = -0.023$, whereas our value is $p_p = -0.030$, in full

agreement with Eq. (6). It is therefore the combined effects of the underestimation of the proton contribution and of the absence of binding effects, which originate the shift upward of g_1^3 at $x \simeq 0$ and the flattening of $A_{3\vec{\mathrm{He}}}$ at $x \sim 1$ exhibited by the results of Ref. [8] with respect to our ones. In closing, we shall consider the first moment of the ^{3}He spin structure function, viz $\Gamma_3 = \int_0^1 g_1^3(x)dx$. It can readily be shown that, provided the Bjorken sum rule [18] holds and the assumption (4) is valid, one has, independently of the form of $g_1^{p(n)}$:

$$\Gamma_3 = \int_0^1 g_1^3(x)dx =$$
$$= [p_n + 2p_p]\Gamma_p - \frac{1}{6}\frac{g_A}{g_V}\left[1 - \frac{\alpha_s}{\pi}\right]p_n \tag{19}$$

where $\Gamma_p = \int_0^1 g_1^p(x)dx$. Using the values (6) and (7), $g_A/g_V = 1.259$ [19] and $\alpha_s = 0$, one gets $\Gamma_3 = -0.180 + 0.804\Gamma_p$ (if the EMC result [3] is used for Γ_p (Γ_p=0.126), then $\Gamma_3 = -0.079 \pm 0.003$, the error being due to the uncertainties on the values p_p and p_n (cf. Eqs. (6) and (7) (the error generated by Fermi motion and binding is very small: using the series expansion for g_1^3 and changing Λ_N/M by a factor of 15, changes Γ_3 by less than 5%). The new experiments will provide both Γ_p and Γ_3, and the validity of Eq. (19) could be checked: strong deviations of it from the value $-0.180 + 0.804\Gamma_p$ can be interpreted as evidence of the violation of the Bjorken sum rule. We have checked that various relativistic normalizations of the spectral function affect neither g_1^3 nor Γ_3.

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FIGURES

FIG. 1.   (a) The ^{3}He asymmetry (Eq. (1)) calculated within the convolution approach (Eq. (15)(full)). Also shown are the neutron (short dash) and proton (long dash) contributions. (b) The SSF g_1^3 of ^{3}He (full); also shown are the neutron (short dash) and proton (long dash) contributions. The dotted curve represents the free neutron structure function g_1^n. The difference between the dotted and short–dashed lines is due to nuclear structure effects.

FIG. 2.   (a) The ^{3}He asymmetry calculated with different nuclear models. Dotted line: no nuclear effects (Eq. (3)); short-dashed line: S' and D waves of ^{3}He taken into account (Eq. (5)); long-dashed line: S' and D waves of ^{3}He taken into account plus Fermi motion effects; full line: S' and D waves of ^{3}He taken into account plus Fermi motion and binding effects. (b) The same as in (a) but for the SSF g_1^3 of ^{3}He. The long–dashed curve is hardly distinguishable from the full one and it is not reported.

FIG. 3.   The free neutron structure function g_1^n (dots) compared with the neutron structure function given by Eq. (17)(dash). The difference between the two curves is due to Fermi motion and binding effects.